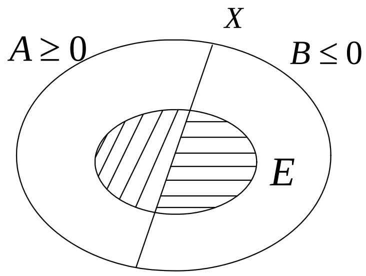
Class 10



Thm 2. u signed measure

Then $\exists$ measures u_1, u_2, one is finite $\ni$ $u = u_1 - u_2$.

Pf: Let $u^+(E) = u(E \cap A)$ *for* $E \in \boldsymbol{a}$

$u^-(E) = -u(E \cap B)$

Then (1) $u^+, u^-: \boldsymbol{a} \to [0, \infty]$

(2) $u^+(\phi) = u(\phi) = 0$

$u^-(\phi) = -u(\phi) = 0$

(3) $u^+(\bigcup_i E_i) = u((\bigcup_i E_i) \cap A) = u(\bigcup_i (E_i \cap A)) = \sum_i u(E_i \cap A) = \sum_i u^+(E_i)$

for disjoint $\{E_i\}$. Also for u^-.

(4) $u^+(X) = u(A)$

$u^-(X) = -u(B)$

$\Rightarrow$ one of them finite

(Ex. In proof of Thm 1, $-\infty < u(B) \leq 0$).

(5) $u^+(E) - u^-(E) = u(E \cap A) + u(E \cap B) = u(E)$

$\therefore u = u^+ - u^-$

Note: 1. In general, μ_1, μ_2 not unique. (Ex. $\mu = \mu_1 - \mu_2 = (\mu_1 + \mu_0) - (\mu_2 + \mu_0)$).

2. $u = u^+ - u^-$ Jordan decomposition of u

3. Jordan decomposition indep. of Hahn decomposition. (Ex.1.10.4)

Def. u^+ upper variation of u

u^- lower variation of u

$|u| = u^+ + u^-$ total variation of u. ($|u|$ measure)

u finite if $|u|$ finite measure

u σ-finite if $|u|$ σ-finite measure

Homework: Ex.1.10.3, 1.10.4

Analogue:

1. $a = a^+ - a^-$, where $a^+ = \frac{1}{2}(|a| + a) \geq 0$ & $a^- = \frac{1}{2}(|a| - a) \geq 0$

2. $|a| = a^+ + a^-$

3. $\forall$ $a \in [-\infty, \infty]$, $a = a_1 - a_2$, where $a_1, a_2 \in [-\infty, \infty]$, one is finite.

4. a_1, a_2 not unique.